Three-dimensional galvanic distortion of three-dimensional regional conductivity structures: Comment on “Three-dimensional joint inversion for magnetotelluric resistivity and static shift distributions in complex media” by Yutaka Sasaki and Max A. Meju

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[1] Distortion of natural time variations of the electric and magnetic fields induced by external magnetic sources by local, near-surface inhomogeneities has been the bane of the magnetotelluric method (MT). These distortions are caused by galvanic charges on conductivity gradients at the boundaries of near-surface inhomogeneities that are below the resolution of the MT experiment. They often dominate over the inductive response of deeper structures, leading to erroneous interpretations if not appropriately considered. Indeed, one can argue that MT has become a generally robust and useful geological mapping tool only with the advent of distortion recognition, appraisal and removal methods. On land, these distortions are usually low in regions of laterally uniform surficial layers, such as sedimentary basin environments, and extreme in highly heterogeneous resistive environments, such as on Precambrian regions. On the seafloor they are usually extreme. They are always present and affect the MT responses derived from the observed fields to a greater or lesser extent, and must be considered in any analyses and subsequent interpretations.

[2] Distortion methods for MT data responding to onedimensional (1-D) and two-dimensional (2-D) structures (reviewed below) have been advanced and are usually quite effective, except in the situations of very severe distortion. For data from three-dimensional (3-D) structures however, the problems caused by these galvanic charges becomes far more complex and require innovative analysis and treatment.

[3] In their paper, Sasaki and Meju [2006] describe their approach to 3-D inversion of MT data for 3-D structure taking galvanic distortion, which they characterize and simplify as static shifts, into account. Their paper is an update on prior work by Sasaki [2004] with the same approach. The assumption made by the authors is that the magnitudes of the observed off-diagonal elements of the impedance tensor can be described as geometrically shifted versions of their true values, namely in their terms

\[ d_o = d + Gs, \]

where \( d_o \) is the vector of observed shifted log (apparent resistivities) and phases, \( d \) is the vector of unshifted log (apparent resistivities) and phases, \( G \) is a matrix relating to static shifts of the data, and \( s \) is a vector of shift parameters. As expressed by Sasaki and Meju [2006, p. 4], “rows of \( G \) corresponding to phases are zeros, because they are not affected by static shift.” This supposition is equivalent to assuming that the elements of the observed impedance tensor, \( Z_{obs} \), are related to the true, unshifted elements by simple, frequency-independent, geometric multipliers \( c_{ij} \):

\[ Z_{obs} = \begin{pmatrix} c_{xx}Z_{xx} & c_{xy}Z_{xy} \\ c_{yx}Z_{xy} & c_{yy}Z_{yy} \end{pmatrix}, \]

where \( Z_{ij} \) are the true elements (dependence on frequency assumed). The purpose of this comment is to draw attention to the fact that this form is valid only in two very restrictive cases; either that the structures are 2-D (which begs the question as to why 3-D inversion is being undertaken) or in the unlikely case that the distortion at every observation site is such that there are only statistically significant values on the diagonal elements of the distortion tensor, and the off-diagonal distortion elements are statistically zero.

[4] In addition, this comment draws attention to the point that one cannot adopt a technique routinely applied in 2-D inversion of MT data to deal with static shifts, namely to set large errors floors on the apparent resistivities, as the phases are also distorted from their true regional values. The elements in the distorted impedance tensor are amplitude and phase mixtures of the true impedance tensor elements, not amplitude mixtures alone.

[5] A tensor-based general method for correcting galvanic distortion was first proposed for the 1-D problem by Larsen [1977], building on the work of Berdichevsky and Dmitriev and their coworkers and students on recognizing surficial distortions [Berdichevsky and Dmitriev, 1976]. The approach...
was extended to two dimensions first by Richards et al. [1982] then Bahr [1984, 1988, 1991], Bailey and Groom [1987], and Groom and Bailey [1989, 1991], among others. More recently, two approaches for dealing with galvanic distortion for 3-D regional structures have been proposed by Garcia and Jones [1999, 2002] and Utda and Munekane [2000]. Ledo et al. [1998] proposed a hybrid method for treatment that may be applicable in some situations, where the highest frequencies can be considered responsive to 3-D distortion of 2-D structures, and conventional 2-D galvanic approaches used, then the derived distortion terms applied at the lower frequencies to remove their effects from the 3-D data.

[6] As shown by a number of authors, the electric effects of galvanic distortion can be represented by a real 2 × 2 tensor that operates on the regional impedance tensor:

\[
\mathbf{Z}_{\text{obs}}(\omega) = \mathbf{C} \cdot \mathbf{Z}_{\text{reg}}(\omega),
\]

where \( \mathbf{Z}_{\text{obs}} \) is the observed 2 × 2 complex, frequency-dependent, MT impedance tensor, \( \mathbf{Z}_{\text{reg}} \) is the true regional 2 × 2 complex, frequency-dependent, MT impedance tensor, \( \omega \) is the radian frequency, and \( \mathbf{C} \) is a 2 × 2, frequency-independent, real tensor given by [Groom and Bailey 1989, 1991; Chave and Smith 1994]

\[
\mathbf{C} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.
\]

The magnetic effects of galvanic distortion, discussed by Chave and Smith [1994] and Chave and Jones [1997], among others, become rapidly negligible with increasing period, and will not be treated here as they are almost universally neglected for land-based MT studies. (They are, however, highly significant on the seafloor and cannot be neglected for marine MT studies, and indeed also marine CSEM with a dipole receiver.)

[7] In 1-D, the regional impedance tensor adopts an anti-diagonal form, with opposite phase on the elements to account for the right hand rule [Larsen 1977] (dependence on frequency of impedances assumed):

\[
\mathbf{Z}_{\text{obs}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & Z_{\text{xy}} \\ -Z_{\text{yx}} & 0 \end{pmatrix} = \begin{pmatrix} -bZ_{\text{xy}} & aZ_{\text{xy}} \\ -dZ_{\text{yx}} & cZ_{\text{yx}} \end{pmatrix}.
\]

Clearly the observed impedance phases are unaffected, and the impedances themselves are scaled by real numbers. This scaling produces a frequency-independent geometrical shift of the apparent resistivity curves, which has become known as static shifts [Jones 1988; Sternberg et al., 1988].

[8] Distortion of electric fields for 2-D regional electric fields was first suggested by Richards et al. [1982], and in the strike coordinates of the 2-D structures the observed impedances are given by

\[
\mathbf{Z}_{\text{obs}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & Z_{\text{xy}} \\ Z_{\text{yx}} & 0 \end{pmatrix} = \begin{pmatrix} bZ_{\text{yx}} & aZ_{\text{yx}} \\ dZ_{\text{yx}} & cZ_{\text{yx}} \end{pmatrix}.
\]

The problem comes when the data are acquired in an arbitrary reference frame, and the geoelectric strike, \( \theta \), must be recovered from the data themselves acquired in a rotated reference frame:

\[
\mathbf{Z}_{\text{obs}}(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & Z_{\text{xy}} \\ Z_{\text{yx}} & 0 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.
\]

[9] Note that at a single frequency this problem is inherently underdetermined. We have nine unknowns (\( \theta, a, b, c, d \), and the real and imaginary parts of the true 2-D complex impedance elements \( Z_{\text{xy}} \) and \( Z_{\text{yx}} \)), but only eight known (the real and imaginary parts of the four complex elements of the observed tensor). In the 1960s and 1970s a number of amplitude-based techniques were proposed to recover strike angle, such as Swift’s strike [Swift, 1967], but these fail calamitously in the presence of distortion (as do amplitude-based measures of dimensionality, like Swift’s skew [Swift, 1967]). Taking the consequences of the insightful paper by Richards et al. [1982] further, Bahr [1984, 1988] showed that the correct geoelectric strike could be obtained by considering the rotational properties of the observed impedance tensor, and, in particular, finding the coordinate direction \( \theta \) in which the elements in the columns of the tensor have equal phase, i.e., \( \varphi_{\text{yx,obs}}(\theta) = \varphi_{\text{yx,obs}}(\theta) \) and simultaneously \( \varphi_{\text{xy,obs}}(\theta) = \varphi_{\text{yx,obs}}(\theta) \). Using the model of galvanic distortion described by equation (7), Bailey and Groom [1987] and Groom and Bailey [1989, 1991] undertook a physical and mathematical treatment of the problem of estimating the regional 2-D impedances, and used a tensor decomposition approach to devolve the problem into determinable and indeterminable parameters, and to solve for the seven determinable parameters.

[10] Assuming that the strike direction \( \theta \) can be correctly recovered, then the off-diagonal elements of the observed tensor are correctly those of the true 2-D tensor \( Z_{\text{xy}} \) and \( Z_{\text{yx}} \) scaled by the galvanic scaling factors \( a \) and \( d \) respectively. So once again in the strike direction there is an unknown real scaling factor for each apparent resistivity curve, \( \alpha^2 \) and \( \beta^2 \) for \( p_{\text{xx}}, \) and \( p_{\text{yx}}, \) respectively, but the phases \( \varphi_{\text{yx}} \) and \( \varphi_{\text{yx}} \) are unaffected. Many methods have been proposed to estimate those static shift scaling factors for 1-D and 2-D regional structures, and their use, determination, application and implementation for 3-D regional structures is the issue of this comment.

[11] In fully 3-D, the problem becomes far more complex. The observed impedance tensor is given by the galvanic distortion of a full 2 × 2 complex tensor describing the regional 3-D structures:

\[
\mathbf{Z}_{\text{obs}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Z_{\text{xx}} & Z_{\text{xy}} \\ Z_{\text{yx}} & Z_{\text{yy}} \end{pmatrix} = \begin{pmatrix} aZ_{\text{xx}} + bZ_{\text{yx}} & aZ_{\text{yx}} + bZ_{\text{yy}} \\ cZ_{\text{yx}} + dZ_{\text{xx}} & cZ_{\text{yy}} + dZ_{\text{yx}} \end{pmatrix}.
\]
more than by simple frequency-independent, geometric shifting, but also are the phases—there is phase mixing in all elements.

[12] For the off-diagonal terms of the observed impedance tensor, which are usually those treated with more importance even in the 3-D case, then

\[
\mathbf{Z}_{xy,\text{obs}} = a \mathbf{Z}_{xx} + b \mathbf{Z}_{yy},
\]

\[
\mathbf{Z}_{yx,\text{obs}} = c \mathbf{Z}_{xx} + d \mathbf{Z}_{yy}.
\]

(9)

Computing the apparent resistivities from these distorted impedances reveals that the apparent resistivities are not simply shifted versions of the true off-diagonal apparent resistivities:

\[
\rho_{a,xy,\text{obs}} = \frac{1}{\omega^2} |\mathbf{Z}_{xy,\text{obs}}|^2
\]

\[
= \frac{1}{\omega^2} |a \mathbf{Z}_{xx} + b \mathbf{Z}_{yy}|^2
\]

\[
= \frac{1}{\omega^2} \left( a^2 |\mathbf{Z}_{xx}|^2 + ab \left( \mathbf{Z}_{xx} \mathbf{Z}_{yy}^* + \mathbf{Z}_{yy} \mathbf{Z}_{xx}^* \right) + b^2 |\mathbf{Z}_{yy}|^2 \right)
\]

\[
= \frac{1}{\omega^2} \left( a^2 |\mathbf{Z}_{xx}|^2 + 2ab \left( \mathbf{R}_{xy} \mathbf{I}_{xy}^* + \mathbf{I}_{xy} \mathbf{R}_{xy}^* \right) + b^2 |\mathbf{Z}_{yy}|^2 \right)
\]

\[
= \frac{1}{\omega^2} \left( a^2 |\mathbf{Z}_{xx}|^2 + 2ab \left( |\mathbf{Z}_{xy}|^2 \cos \phi_{xy} \cos \phi_{yx} + |\mathbf{Z}_{yy}|^2 \sin \phi_{xy} \sin \phi_{yx} \right) + b^2 |\mathbf{Z}_{yy}|^2 \right)
\]

(10)

and similarly

\[
\rho_{a,yx,\text{obs}} = a^2 \rho_{a,xy} + 2ab \sqrt{\rho_{a,xy} \rho_{a,yx}} \cos (\phi_{yx} - \phi_{xy}) + b^2 \rho_{a,yy}.
\]

where \(R_{ij}\) and \(I_{ij}\) are the real and imaginary parts of the undistorted \(Z_{ij}\) impedance, and \(\phi_{ij}\) is its phase. These observed (distorted) apparent resistivities include amplitude-distorted versions of the diagonal terms plus an amplitude-distorted mixing of the two, governed by their phase differences.

[13] For the phases, the true undistorted phases of the off-diagonal elements are given by

\[
\phi_{xy} = \tan^{-1} \left( \frac{I_{xy}}{R_{xy}} \right) \quad \text{and} \quad \phi_{yx} = \tan^{-1} \left( \frac{I_{yx}}{R_{yx}} \right).
\]

(11)

whereas the phases of the observed (distorted) off-diagonal elements are

\[
\phi_{xy,\text{obs}} = \tan^{-1} \left( \frac{aI_{xy} + bI_{yx}}{aR_{xy} + bR_{yx}} \right)
\]

\[
\phi_{yx,\text{obs}} = \tan^{-1} \left( \frac{dI_{yx} + cI_{xy}}{dR_{yx} + cR_{xy}} \right).
\]

(12)

Thus, the observed off-diagonal elements are each a mixture of the two elements in each column of the true 3-D regional tensor. Note that in the 1-D case, and the 2-D case when in the strike angle, then \(\rho_{a,xy} = \rho_{a,xx} = 0\), and equation (10) reduces to \(\rho_{a,xy,\text{obs}} = a^2 \rho_{a,xy}\) and \(\rho_{a,yx,\text{obs}} = a^2 \rho_{a,yx}\), and equation (12) for the phases reduces to the correct forms of equation (11), i.e., galvanic distortion introduces simple geometrical static shifts by factors of \(a^2\) and \(d^2\) respectively in the apparent resistivities, without any phase effects. In 3-D case in 2-D if the Groom-Bailey twist and shear distortion parameters were both zero everywhere, which never occurs for real data.

[15] The other case where this occurs is when the regional impedance tensor does not have any diagonal terms, i.e., \(Z_{xx} = Z_{yy} = 0\), which is true when the regional structures are 2-D. That then begs the question as to why 3-D inversion is being invoked in the first place.

[16] There is a pathological third case, and that is that the site is located at a point of symmetry above a laterally symmetrical 3D body (essentially, the center of a circular column), and in this case the diagonal terms are zero and the approach will work. However, this will be true for one site at most in a profile or array of observation locations.

[17] It is important to recognize that the amplitude and phase mixing in equations (8), (10) and (12) means that another technique routinely used in 2-D inversion to deal with static shifts, namely to assign high error floors to the apparent resistivity data, is also inappropriate for galvanic distortion of 3-D data. Zhdanov et al. [2011, p. 3] recently published a paper in which this was exactly how they dealt with galvanic distortion: “We have reduced the static shift effect by normalizing the observed MT impedances with their absolute values, which effectively resulted in the phase inversion of the impedances. It is well known that the phases are less sensitive to the galvanic distortions, caused by near-surface inhomogeneities.”

[18] This is a correct approach and correct statement only in the 2-D case. It is incorrect in the 3-D case. Yes, assigning
large errors bars will deal with the mixing in the amplitudes (equation (10)), but the observed phases are mixtures of the regional phases (equation (12)), so will be incorrectly modeled.

[19] Another correction technique used in MT to deal with static shifts is making controlled source EM measurements at the MT site, typically using a transient magnetic technique (loop-loop) [Sternberg et al., 1988; Pellerin and Hohmann, 1990; Meju, 1996] or using DC resistivity [Spitzer, 2001; Meju, 2005], and shifting the MT apparent resistivity curves to the levels from those CSEM experiments. However, the EM fields associated with regional MT current systems will not generally have the same geometry as current systems from local, small-scale controlled sources, except for a uniform half-space. Thus, these methods are applicable at high frequencies on regions that display relatively homogenous upper layers laterally, such as sedimentary basins, but are questionable in regions of highly heterogeneous surficial layers where distortion is likely to be strong. In addition, it is unclear how one obtains the “correct” levels of the diagonal elements $Z_{xx,obs}$ and $Z_{yy,obs}$ from this approach.

[20] Just how important and significant is this? Clearly, distortion tensors that are only diagonal in form will be extremely rare — there will always be some component of distortion, however small, that has to be addressed. If distortion is not addressed, in order to fit the data to within its limits, galvanic distortion will appear as structure in the model. As is obvious in equations (8), (10) and (12), the amplitude and phase mixing caused by the distortion is important for the off-diagonal impedances only when the distorted diagonal terms become as large as the distorted off-diagonal terms, i.e., the magnitude of $bZ_{xy}$ and/or $cZ_{xx}$ becomes of order $dZ_{xy}$ and/or $dZ_{xx}$ becomes of order $dZ_{yy}$. This will occur either when the distortion is severe, or when the diagonal terms are large, i.e., significant 3-D structure. One problem to note is that whereas $b$ and $c$ are frequency-independent (in the galvanic limit), $Z_{xx}$ and $Z_{yy}$ are not, and the distortion terms in equation (10) will display frequency dependence. There may be frequencies when the 3-D structures respond in a manner that appear 2-D, to within the errors of the data, so the 2-D distortion approaches are valid, but there will be others when the 3-D effects result in significant diagonal terms.

[21] However, for the diagonal terms of the observed (distorted) impedance tensor, then the typically smaller (undistorted) diagonal terms will be overwhelmed by the much larger distorted off-diagonal terms even in the case of weak distortion, i.e., the magnitude of $bZ_{xx}$ overwhelsm $aZ_{xx}$ and/or $cZ_{yx}$ overwhelmed $dZ_{yx}$. It is precisely these diagonal terms that offer the greatest increase in resolution of 3-D geometries afforded by moving from 2-D inversion to 3-D inversion, and it is precisely these terms that are most affected by distortion.

[22] Three-dimensional inversion of MT data is becoming more commonplace, especially as now there is a publicly available code thanks to Weerachai Sirirunvaraporn [Sirirunvaraporn et al., 2005]. The situation now is comparable to that which existed in the late 1980s and early 1990s, when freely available 2-D inversion codes, lead by Stephen Constable [de Groot-Hedlin and Constable, 1990], had a huge impact on MT interpretations and caused a quantum leap in MT resolution of Earth structures by allowing MT data to be inverted in 2-D rather than in 1-D. There are issues though in 3-D inversion related to the size of the models, given the very high memory requirements and long computing times; typical 2-D models of 200 horizontal cells and 100 vertical cells still offer superior resolution where appropriate. 3-D inversions will become routine, as faster processors and faster codes become available. However, as was the case with 2-D inversion that took a significant leap forward when distortion effects were recognized and appropriate techniques developed, so does the 3-D inversion of MT data require such attention. Given the difficulties of applying an equivalent approach in 3-D as used in 2-D of identifying and removing distortion effects shown by Garcia and Jones [1999, 2002], a preinversion analysis step may not be the most fruitful. Some are approaching this problem by allowing the surficial layer to be highly heterogeneous [Patro and Egbert, 2011]. The most optimum approach though is probably to consider the frequency-independent galvanic distortion factors as four more unknowns at each site that have to be solved for, as was done in 2-D by de Groot-Hedlin [1995]. Such an approach is being pursued by Miensopust [2009] and Avdeeva et al. [2011] independently, and both are showing promise.

References


