

Comment on 'Geomagnetic Depth Sounding by Induction Arrow Representation: A Review' by G. P. Gregori and L. J. Lanzerotti

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In this recent review, *Gregori and Lanzerotti* [1980] address themselves to the problem of describing the relationships between various 'induction vectors' as proposed by different induction workers. This in itself is an admirable exercise and certainly of great worth to the geomagnetic community, both for those studying induction effects and for those using ground-based magnetometers to determine ionospheric and magnetospheric processes. However, certain assumptions were made, but not stated, by the authors when comparing various 'arrows,' and it is the purpose of this comment to state more explicitly some of those limiting assumptions and to give a more general relationship between time domain determined arrows and frequency domain determined ones.

Gregori and Lanzerotti state that the frequency domain real Schmucker arrow [*Schmucker*, 1970] is related to the time domain Wiese arrow [*Wiese*, 1962] by

$$\mathbf{v}_s = -\mathbf{v}_w \quad (1)$$

(their equation (11)).

The general Schmucker vector is complex and is derived from the inductive transfer functions given by

$$Z_a(\omega) = z_H(\omega) \cdot H_n(\omega) + z_D(\omega) \cdot D_n(\omega) + \mathcal{E} \quad (2)$$

where $Z_a(\omega)$ is the anomalous vertical magnetic field, $H_n(\omega)$ is the normal (i.e., in the absence of a lateral variation in electrical conductivity) northward component of the horizontal magnetic field, $D_n(\omega)$ is the normal eastward horizontal magnetic field component, \mathcal{E} is the error of misfit, and [$z_H(\omega)$, $z_D(\omega)$] are the inductive transfer functions at frequency ω . For single-station data, derivation of (z_H , z_D) given by (2) is not possible, and hence those transfer functions that relate the total measured horizontal fields to the total measured vertical fields, as given by

$$Z(\omega) = A(\omega) \cdot H(\omega) + B(\omega) \cdot D(\omega) + \mathcal{E}' \quad (3)$$

where $H(\omega)$, $D(\omega)$, and $Z(\omega)$ are the total fields, are often determined. In order to interpret (A , B) it is usually assumed that

$$z_H(\omega) \approx A(\omega) \quad z_D(\omega) \approx B(\omega)$$

and the assumptions under which these approximations are reasonably valid are detailed in many works, for example, *Alabi et al.* [1975].

Inverse Fourier transformation of (3) yields

$$z(t) = a(t) * h(t) + b(t) * d(t) + \mathcal{E}'' \quad (4)$$

where the asterisk denotes the convolution operation, $h(t)$, $d(t)$, and $z(t)$ are the measured time variations of the three orthogonal components of the magnetic field, and [$a(t)$, $b(t)$] are the impulse response functions of the two-input/single-output linear system described by (4).

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If the data consist of a single-frequency component ω_0 , which may be natural (i.e., pulsation data) or artificial (i.e., after filtering), then the field components may be expressed as

$$z(t) = z_0 \cos(\omega_0 t) \quad (5a)$$

$$h(t) = h_0 \cos(\omega_0 t + \delta_h) \quad (5b)$$

$$d(t) = d_0 \cos(\omega_0 t + \delta_d) \quad (5c)$$

after a suitable choice of time axis. Substituting equations (5) into (4), the convolution equation reduces to a simple multiplication equation, namely (see, for example, *Schmucker* [1980]),

$$z(t) = \text{Re} [A(\omega_0)] \cdot h(t) - \text{Im} [A(\omega_0)] \cdot h\left(t + \frac{T}{4}\right) + \text{Re} [B(\omega_0)] \cdot d(t) - \text{Im} [B(\omega_0)] \cdot d\left(t + \frac{T}{4}\right) \quad (6)$$

where T is the period of interest, that is, $T = 2\pi/\omega_0$. In terms of the amplitude and phase of the transfer functions at frequency ω_0 , (6) may be expressed as

$$z(t) = a_{\omega_0} h_0 [\cos(\omega_0 t + \delta_h) \cos(\omega_0 \tau_h) - \sin(\omega_0 t + \delta_h) \sin(\omega_0 \tau_h)] + b_{\omega_0} d_0 [\cos(\omega_0 t + \delta_d) \cos(\omega_0 \tau_d) - \sin(\omega_0 t + \delta_d) \sin(\omega_0 \tau_d)] \quad (7)$$

where

$$a_{\omega_0} = |A(\omega_0)| \quad b_{\omega_0} = |B(\omega_0)|$$

$$\tau_h(\omega_0) = \frac{1}{\omega_0} \tan^{-1} \left(\frac{\text{Im} [A(\omega_0)]}{\text{Re} [A(\omega_0)]} \right)$$

$$\tau_d(\omega_0) = \frac{1}{\omega_0} \tan^{-1} \left(\frac{\text{Im} [B(\omega_0)]}{\text{Re} [B(\omega_0)]} \right)$$

Using standard trigonometrical expressions and employing the field expressions detailed in (5a)-(5c), equation (7) may be written as

$$z(t) = a_{\omega_0} \cdot h[t + \tau_h(\omega_0)] + b_{\omega_0} \cdot d[t + \tau_d(\omega_0)] \quad (8)$$

which indicates that it is possible to interpret the impulse response functions as spikes at τ_h and τ_d , respectively, where the latter two terms representing the lead, or lag, of the horizontal components with respect to the vertical component.

If the induction processes are such that there is pure self-induction, then the inducing and induced fields are totally in phase, giving purely real values for (A , B) at all frequencies; hence $\tau_h = \tau_d = 0$ for all ω . Only for this special case do (6) and (8) reduce to

$$z(t) = a_{\omega_0} \cdot h(t) + b_{\omega_0} \cdot d(t) \quad (9)$$

where a_{ω_0} and b_{ω_0} are given by the real parts of $A(\omega_0)$ and $B(\omega_0)$, respectively.

The real, or 'in-phase,' Schmucker arrow at frequency ω is defined by

$$v_s(\omega) = \{ \text{Re}^2 [A(\omega)] + \text{Re}^2 [B(\omega)]^{1/2} \} \tan^{-1} \left(\frac{-\text{Re} [A(\omega)]}{\text{Re} [A(\omega)]} \right) \quad (10)$$

Hence for single-frequency data with pure self-induction, (10) becomes

$$v_s(\omega_0) = (a_{\omega_0}^2 + b_{\omega_0}^2)^{1/2} \tan^{-1} (-b_{\omega_0}/a_{\omega_0}) \quad (11)$$

and thus may be estimated in the time domain from application of (9).

The Wiese vector [Wiese, 1962] is given by a regressive fit of a first-order polynomial of $\Delta H/\Delta Z_{\text{extr}}$ on $\Delta D/\Delta Z_{\text{extr}}$, where ΔZ_{extr} denotes an extremum value of ΔZ , the disturbed vertical magnetic field, and ΔH and ΔD are those values of the disturbed horizontal field components at the time that ΔZ attains the extremum. This is expressed mathematically by

$$\Delta Z_{\text{extr}}(t_{\text{extr}}) = a \cdot \Delta H(t_{\text{extr}}) + b \cdot \Delta D(t_{\text{extr}}) \quad (12)$$

(see, for example, Untiedt [1970, equation (9)]).

It can be immediately recognized that expression (9) is identical to expression (12). The Wiese vector is given by

$$v_w = (a^2 + b^2)^{1/2} \tan^{-1} (b/a) \quad (13)$$

and by comparison with (11), it is obvious that

$$v_s(\omega_0) = -v_w$$

for the limiting case of single-frequency data and pure self-induction.

A less restrictive relationship between the Schmucker and the Wiese arrows has been given by Schmucker [1980] which only necessitates assuming that the phase lags, or leads, of the horizontal components with respect to the vertical component are equal, that is, $\tau_h = \tau_d$.

At $t = 0$, a time when $z(t)$ attains an extremum, the field components, from equations (5), are

$$z(0) = z_0 \quad (14a)$$

$$h(0) = h_0 \cos(\delta_h) \quad (14b)$$

$$d(0) = d_0 \cos(\delta_d) \quad (14c)$$

and the relationship between them, as described by (7), becomes

$$z(0) = a_{\omega_0} h_0 [\cos(\delta_h) \cos(\omega_0 \tau_h) - \sin(\delta_h) \sin(\omega_0 \tau_h)] \\ + b_{\omega_0} d_0 [\cos(\delta_d) \cos(\omega_0 \tau_d) - \sin(\delta_d) \sin(\omega_0 \tau_d)] \quad (15)$$

At a time one quarter of a cycle later, that is, $t = \pi/2\omega_0$, $z(t)$ passes through zero, and hence from (7),

$$z(\pi/2\omega_0) = 0 = a_{\omega_0} h_0 [-\sin(\delta_h) \cos(\omega_0 \tau_h) - \cos(\delta_h) \sin(\omega_0 \tau_h)] \\ + b_{\omega_0} d_0 [-\sin(\delta_d) \cos(\omega_0 \tau_d) - \cos(\delta_d) \sin(\omega_0 \tau_d)] \quad (16)$$

Assuming $\tau_h = \tau_d = \tau$, multiplying (15) by $\cos(\omega_0 \tau)$, multiplying (16) by $\sin(\omega_0 \tau)$, and subtracting the latter from the former, gives

$$\cos(\omega_0 \tau) \cdot z(0) = a_{\omega_0} h_0 \cos(\delta_h) + b_{\omega_0} d_0 \cos(\delta_d) \quad (17)$$

Substituting (14b) and (14c) into (17) gives

$$\cos(\omega_0 \tau) \cdot z(0) = a_{\omega_0} h(0) + b_{\omega_0} d(0) \quad (18)$$

where time $t = 0$ represents a time of an extremum in $z(t)$.

By comparing (18) with (12) it is apparent that the Wiese vector is related to the real Schmucker vector by

$$v_s(\omega_0) = -v_w / \cos(\omega_0 \tau) \quad (19)$$

for the case of single-frequency data and $\tau_h = \tau_d$. This latter restriction is equivalent to requiring that the real and imaginary Schmucker vectors, as defined by Schmucker [1970, equation (3.19)], are either exactly in the same direction or exactly opposite in orientation, that is, 0° or 180° angle between them. This restriction is upheld for all two-dimensional anomalies but rarely for the more general three-dimensional anomalies.

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