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Letter to the Editor

On the Equivalence of the "Niblett" and "Bostick" Transformations in the Magnetotelluric Method

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Introduction

In the $1D$ interpretation of magnetotelluric data, it is often useful to discover a reasonable first-approximation to the true conductivity-depth distribution beneath the recording location. This may be undertaken in the field, in order to ascertain if the station spacing is satisfactory or whether a greater station density of coverage is required, or at the base laboratory, as a prelude to a more sophistocated $1D$ inversion of the data.

There are three approximations presently in use by workers whose interest lies in the conductivity structure of the earth: (1) the Schmucker ρ^*-z^* scheme (Schmukker, 1970); (2) the Bostick transformation (Bostick, 1977); and (3) the Niblett approximation (Niblett and Sayn-Wittgenstein, 1960). The first two are in widespread use in western Europe and north America, whilst the Niblett approximation appears to be strongly favoured in eastern Europe and the USSR.

Weidelt et al. (1980) have previously detailed the relationship between Schmucker's ρ^*-z^* and Bostick's transformation; it is the purpose of this letter to demonstrate that Bostick's transformation and Niblett's approximation are *very* equivalent - they give *exactly* the same resistivity-depth profiles!

Theory

Both the Bostick transformation and the Niblett approximation are applied to the derived apparent resistivity curve $\rho_a(T)$ only and, as such, may be considered to be superior to Schmucker's ρ^*-z^* when the phase information does not exist or is thought to be unreliable.

The Bostick transformation and the Niblett approximation give a resistivity-depth distribution, $\rho_B(h)$ and $\rho_N(h)$, where h is a "penetration depth" in a half space medium of resistivity equal to the apparent resistivity at that particular period T, defined by

$$h=\sqrt{\frac{\rho_a(T)T}{2\pi\mu_0}}.$$

Note that this penetration depth implies an attenuation factor of approximately $\frac{1}{2}$ instead of the more usual skin depth attenuation of $1/e$.

The "Bostick" resistivity, $\rho_B(h)$, at depth h is given by

$$\rho_B(h)=\rho_a(T)\frac{1+m(T)}{1-m(T)}$$

where $m(T)$ is the gradient of the apparent resistivity curve on a log-log scale, i.e.

$$m(T)=\frac{d\log(\rho_a(T))}{d\log(T)}=\frac{T}{\rho_a(T)}\,\frac{d\rho_a(T)}{dT}.$$

The "Niblett" transformation gives a conductivity at depth h, $\sigma_N(h)$, from

$$\sigma_N(h)=h\frac{d\sigma_a(T)}{dh}+\sigma_a(T)$$

where $\sigma_a(T)=1/\rho_a(T)$. Obviously

$$\sigma_N(h)=\sqrt{\frac{\rho_a T}{2\pi\mu_0}}\,\frac{d\left(\frac{1}{\rho_a}\right)}{d\sqrt{\frac{\rho_a T}{2\pi\mu_0}}}+\frac{1}{\rho_a}$$

$$=\sqrt{\rho_a T}\,\frac{d\left(\frac{1}{\rho_a}\right)}{d\sqrt{\rho_a T}}+\frac{1}{\rho_a}$$

(dependence of ρ_a on T assumed) which, after differentiating by parts, becomes

$$\sigma_N(h)=\frac{-2T}{\rho_a\left(T+\rho_a\frac{dT}{d\rho_a}\right)}+\frac{1}{\rho_a}.$$

Hence, $\rho_N(h)=1/\sigma_N(h)$ is given by

$$\rho_N(h)=\rho_a(T)\frac{\left(1+\frac{T}{\rho_a}\,\frac{d\rho_a}{dT}\right)}{\left(1-\frac{T}{\rho_a}\,\frac{d\rho_a}{dT}\right)}$$

$$=\rho_a(T)\frac{1+m(T)}{1-m(T)}.$$

Thus, $\rho_B(h)=\rho_N(h)$ for all depths h.

Vanyan et al. (1980) have previously presented a form of the Niblett transformation involving estimation of the gradient of $\log(\rho_a(T))$ against $\log(\sqrt{T})$, which, after simple manipulation, can now be recognised as also exactly equivalent to the Bostick transformation.

An alternative expression for the Bostick resistivity at depth h has been used by various authors (for example, Weidelt et al., 1980; Goldberg and Rotstein, 1982). This form, given by

$$\tilde{\rho}_B(h) = \rho_a(T)\left(\frac{\pi}{2\phi(T)} - 1\right)$$

employs the phase information $\phi(T)$ and is related to the original by Weidelt's "approximate phase" (Weidelt, 1972). The above expression has the advantage of not requiring an estimate of $m(T)$ to be made, but $\tilde{\rho}_B(h) \neq \rho_B(h)$, and hence $\tilde{\rho}_B(h) \neq \rho_N(h)$.

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