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A multi-station magnetotelluric study in southern Scotland – II. Monte-Carlo inversion of the data and its geophysical and tectonic implications

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Summary. A Monte-Carlo inversion procedure is developed and applied to magnetotelluric data from six locations, two of which are in the Midland Valley of Scotland, three in the Southern Uplands, and one in northern England. The method is described in full in respect of one of the six locations to illustrate both the importance of satisfying the phase as well as the amplitude data and the effect of model acceptance level. The electrical resistivity profiles resulting from application of the method indicate that; (a) there is a conducting zone under the Midland Valley at a depth no greater than 12 km, (b) the crust under the Southern Uplands is mainly resistive, (c) there is a conductor at a depth greater than 24 km in this region, and (d) under northern England there is probably a very highly conducting region very close to the surface. A brief discussion of the possible geophysical and tectonic significance of these models follows.

1 Introduction

In presenting the results of a multi-station magnetotelluric investigation in southern Scotland, Jones & Hutton (1978, referred to later as part I) gave a preliminary interpretation of the electrical conductivity structure at certain sites on the basis of limited forward modelling. In that paper they justified the selection of sites for which the observational data was suitable for one-dimensional interpretation and showed that there was a marked lateral variation in conductivity structure in a traverse from the Midland Valley across the Southern Uplands to northern England.

In this paper, a one-dimensional Monte-Carlo inversion procedure is developed and its application to the data from these selected sites – fig. 1, part I – is discussed. The possible geophysical and tectonic significance of the resulting electrical conductivity is considered.

2 Monte-Carlo inversion

2.1 GENERAL COMMENTS

As stated by Jackson (1973), the complete solution to an inverse problem consists of two parts: (1) finding a solution, and (2) representing, in a meaningful way, the degree of non-

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uniqueness of that solution. For most electromagnetic response data, an acceptable model may be found using curve-fitting methods, but on some occasions, the least-squares procedures are not convergent to a solution (Wu 1968). Also these methods do not provide any information regarding the non-uniqueness of the solution. However, an inherent qualitative feeling for the most critical model parameters may be obtained by curve-matching methods.

A technique frequently used in geophysics for investigating the non-uniqueness of a solution is the random search, or Monte-Carlo, procedure. A model is picked at random from the N-dimensional parameter space, corresponding to the N permitted variables of the model, and the theoretical response of the model is compared with the measured response. If certain criteria are satisfied, then the model is accepted. The criteria are usually of a form which imposes a maximum level for the sum of the squares of departures of the theoretical responses from the measured responses or which restricts the departures within pre-defined limits at any of the response values. If the criteria are not satisfied, then the model is rejected. Another model, *not* statistically related to the first, is then selected from the parameter space and the procedure repeated. This random selection of models – whose number is infinite if the variables are continuous – is repeated until either the whole parameter space has been examined or the computing time limit is exceeded. For large N or for continuous variables, the latter of these two situations limits the number of models which can be examined.

If there is only one closed set of acceptable models in the N-dimensional parameter space, then a more efficient search is undertaken by the Hedgehog procedure (Keilis-Borok & Yanovskaya 1967). In this case, the search is implemented initially by a Monte-Carlo procedure, but, once an acceptable solution has been found, the search is then conducted in an organized manner about that solution. Hence, if there are two, or more, enclosed sets of solutions which are unconnected with each other in the parameter space, only one of these will be found and subsequently explored locally.

In seismology, Monte-Carlo and/or Hedgehog inversion procedures have been applied to Rayleigh wave attenuation data (Mills & Fitch 1977, Monte Carlo; Burton 1977, Hedgehog), surface-wave dispersion data (Biswas & Knopoff 1974, Hedgehog) and global seismic velocity profile data (Press 1968, 1970, Monte-Carlo; Anderssen, Worthington & Cleary 1972, Monte-Carlo) and many other problems.

In geomagnetic induction studies, the Hedgehog inversion procedure does not appear to have been employed to date, but Monte-Carlo inversion has been applied by Anderssen (1970), to global electromagnetic induction data, and by Hermance & Grillot (1974), to regional MT data.

A simple Monte-Carlo inversion scheme has now been developed to derive the range of acceptable models permitted by the observed MT responses from the sites labelled in the map of southern Scotland (fig. 1 of part I). It differs in some aspects from that employed by Hermance & Grillot.

2.2 THE METHOD

Initially, a model of 'n' layers was found, by empirical curve-matching, such that the response satisfied the observed apparent resistivity and phase data to within the 95 per cent confidence limits (see part I). If such a model was not found easily, a model whose theoretical response was within as many of the confidence intervals as possible was chosen. The departure of the response of the model from the observed response was obtained from the expression

$$\psi = \sum [\log[\hat{\rho}(f)] - \log[\rho_m(f)]^2 + \sum [\hat{\phi}(f) - \phi_m(f)]^2 \quad (1)$$

where $\hat{\rho}(f)$ and $\hat{\phi}(f)$ are the estimated apparent resistivity (in Ωm) and phase (in radians) at frequency f, and $\rho_m(f)$ and $\phi_m(f)$ are those of the model at f. The logarithm of the apparent resistivity was used because $\hat{\rho}(f)$ – the estimated apparent resistivity – is distributed lognormally, rather than normally, about $\rho(f)$ – the true apparent resistivity – (Bentley 1973). A new model, whose response was to be examined, was generated by perturbing the parameters of the initial model in a random manner. The resistivities and the thicknesses of the ith layer of the new model were derived from

$$h_i' = h_i 2^{r_i} \quad (2)$$

$$\rho_i' = \rho_i 10^{r_{i+n}} \quad (3)$$

where ρ_i and h_i are the resistivity and thickness of the ith layer of the initial model. The 'dashed' parameters refer to the new model layers, and r represents a random number.

The random numbers (r_i and r_{i+n}) which generated the new model were derived from the NAG (Numerical Algorithms Groups) subroutine GO5ADF. A sequence of numbers from this subroutine is normally distributed with zero mean and unit variance. Hence, 68 per cent of the h_i' thicknesses generated were between $h_i/2$ and $2h_i$ and 68 per cent of the ρ_i' resistivities generated were in the range $\rho_i/10$–$10\rho_i$.

The theoretical response for the new model was calculated and compared with the measured response. If the theoretical response was within more than a stipulated number of the 95 per cent confidence intervals, then the model was accepted. Otherwise it was rejected. If the model was accepted, the value of ψ' was calculated from (1) by replacing $\rho_m(f)$ with $\rho_m'(f)$ (the apparent resistivity of the response of the new model at frequency f) and $\phi_m(f)$ with $\phi_m'(f)$ (similarly for phase). If $\psi' < \psi$, then the initial model parameters were replaced by those of the latest new model. For $\psi' > \psi$, no action was taken. Hence, the parameters which were perturbed, by equations (2) and (3), to give the new model, related to the best fitting model at that moment.

The procedure was repeated by generating other 'new' models from equations (2) and (3) (note: each call on GO5ADF gives a number statistically independent of any other number given by previous calls, until 10^{77} calls have been made). Tests were incorporated to ensure the thicknesses and resistivities, h_i' and ρ_i', of the new model were between preset limits. These constraints not only ensured that the parameters were physically realisable but also were used to restrict the search to that region of the parameter space where acceptable models were likely to be found. This region was approximately delineated by initial tests with only physically realisable constraints applied (i.e. $1 < \rho_i < 10^4$ Ωm and h_i positive).

It was decided to generate the new model from values chosen out of the continuous distributions of each of the parameters. An alternative method would have been to choose it at random from discrete values of each of the parameters, as adopted by Hermance & Grillot (1974). The procedure chosen ensured that the models were selected as randomly as possible and therefore were not subject to the objection made by Haddon & Bullen (1969) to Press' (1968) Monte-Carlo method. Haddon & Bullen claimed that a misleading predominance of complex models resulted from Press' method because the probability of generating a parametrically simple random walk was small. However, they made the implicit assumption that the points of the random model were generated sequentially (Anderssen *et al.* 1972). This cannot be true for the method outlined above.

Incorporated in the procedure used in this study was a facility for keeping any one, or more, of the layer parameters to its initial value. Hence, the thicknesses could be kept constant and the layer resistivities altered or vice versa.

2.3 EXAMPLE OF THE INVERSION METHOD

2.3.1 *Choice of model*

The first step was to decide on the model, whose acceptable parameters the random search method were to discover. In this context, it must be remembered that, in geophysical data interpretation, it is *axiomatic* to find the simplest model – or models – that satisfies the observed response. There is no justification in finding the acceptable parameters of complicated models if the data are not able to resolve such models. The amplitude data for the six stations (see part I) can be satisfied by suitable two-layer models; however, the phase data demand a three-layer interpretation. Hence, determination of the parameters of three-layer models, whose theoretical responses were close to the observed responses, was the objective of the inversion of the data. Whilst the authors are fully aware that 4-, 5- and even 10-layer models can be interpreted from the data, they feel their data do not warrant such an inversion. In Section 3.2 it is shown that an 8-layer inversion of the data from NEW yields models which can be adequately represented by 3-layers.

2.3.2 *The d-space*

The inversion scheme is illustrated with reference to station BOR (Borland). The three-layer model, A, shown in Fig. 1(b), was used as the initial model to invert station BOR's

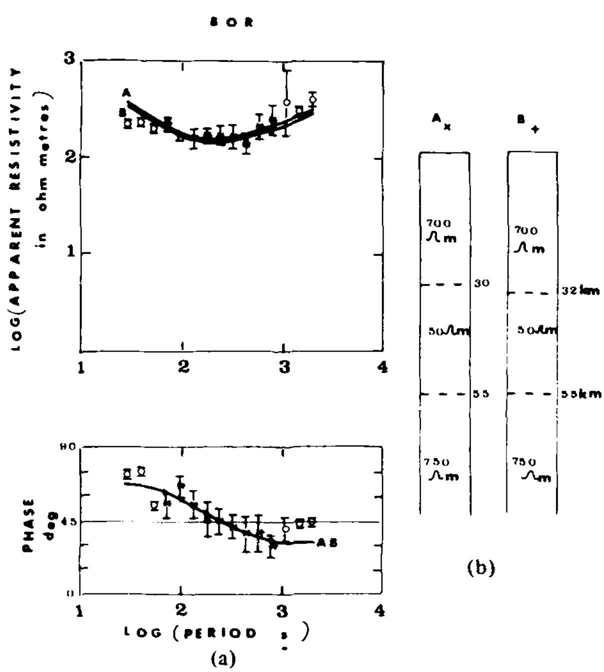


Figure 1. (a) The apparent resistivity and phase data for BOR – the asterisks denote well-estimated values and the open circles the other values. (b) Electrical conductivity models A and B – the initial and final models of the Monte-Carlo inversion of the BOR major data. The theoretical apparent resistivity and phase curves are drawn in (a).

rotated major data (Fig. 1(a)). The theoretical (curves) and observed (asterisks and open circles) responses are also shown in Fig. 1. For the MT data from station BOR, there were nine periods at which the impedances were considered 'well-estimated' (as defined in part I). They are indicated in Fig. 1 by asterisks. Hence, there were 18 confidence intervals, nine for apparent resistivity and nine for phase, which could be employed to constrain the responses of the acceptable models.

In the first trial, the resistivities were held constant at 700 Ωm/50 Ωm/750 Ωm, and the depths to the interfaces were permitted to vary in the range 15–40 km for the top interface d_1, and 25–80 km for the lower interface d_2. Because there were only two variables, the parameter space was two-dimensional, as shown in Fig. 2. This is termed the 'd-space' of the model. The triangle in the top left-hand corner of the d–space is a region from which models were not selected because they would require a negative thickness for the second layer, h_2, and hence were physically untenable. The unshaded region is that in which models were selected at random from the parameter space and their theoretical responses compared with the measured responses. The position of the initial model in the parameter space is indicated by the large solid dot, while the open dots denote models for which the responses were within the 95 per cent confidence limits (see part I) of more than 75 per cent ($0.75 \times 18 = 13.5$, i.e. 14 limits) of the data. The vertical crosses indicate those models for which the responses were within *all* the apparent resistivity confidence intervals (i.e. nine) but an insufficient number of phase intervals (i.e. less than five).

Several features of the parameter d-space are worthy of note:

(1) The most important feature is the obvious reduction in size of the acceptable model region obtained by the inclusion of phase constraints, as summarized in Table 1 below.

(2) The acceptance model region appears to be exclusive, in that any model chosen within its bounds is acceptable at the 75 per cent level. This suggested that it might be more

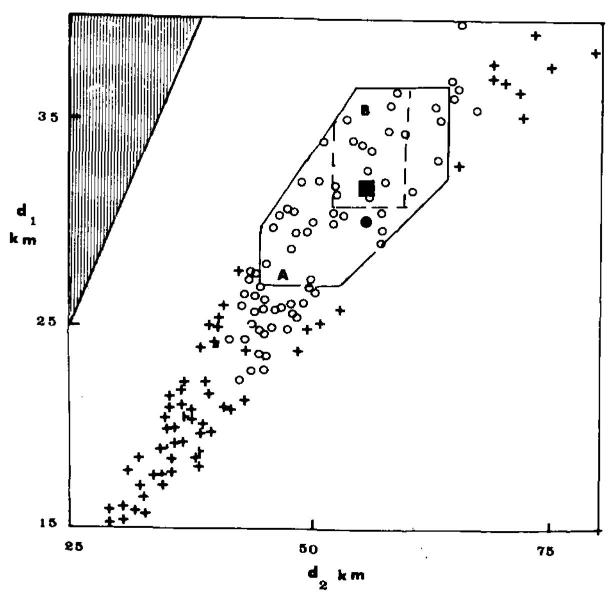


Figure 2. The d-space for the Monte-Carlo inversion of data from BOR, in the Southern Uplands. Open circles denote values of d_1 and d_2 at the 75 per cent acceptance level and the unshaded region is the d-space in which other models were situated. The boundaries A and B denote the areas in which the d values were accepted at 89 and 94 per cent levels respectively.

Table 1. Permitted layer dimensions.

	Amplitude data only satisfied	Amplitude and phase data satisfied
Depth of uppermost interface	?	> 23 km
Thickness of second layer	14–42 km	16–30 km

efficient to explore this two-dimensional space by the Hedgehog method. Since, however, it was intended to generalize the inversion technique to an N-dimensional search procedure, it could not be assumed that there would only be one closed acceptance region in N-dimensional parameter space.

(3) It is apparent from the parameter d-space illustrated in Fig. 2 that the search region was overconstrained since the maximum permissible top layer thickness was not determined.

(4) The model indicated by the black square was that model of those examined, for which the theoretical response was closest, in a least-squares sense, to the observed response, i.e. its ψ' estimated from equation (1) was smallest. This is model B of Fig. 1(b). Its theoretical response is shown by vertical crosses in Fig. 1(a). This Monte-Carlo procedure thus finds not only the bounds of the acceptance model parameters but also an 'optimum' model.

(5) When the acceptance level was increased from 75 to 89 per cent (i.e. 16 of the 18 intervals), only those models whose layer depths were within region A of Fig. 2 had acceptable theoretical responses, and when it was further increased to 94 per cent (17 intervals minimum), the region of acceptable models was further reduced in size to zone B. It is interesting to note that the initial model was not acceptable at the 94 per cent level.

2.3.3 The S-space

The initial test discussed in the previous section showed that the method was viable and could produce seemingly meaningful results. In the second stage the best-fitting model from stage one of the method (the black square of Fig. 2) was employed as the initial model in a random search procedure for a three-layer model with four variable parameters – the resistivity and thickness of the top two layers. The base layer resistivity was kept constant at 750 Ωm. To overcome the major difficulty in multi-variate inversion studies of displaying the parameter space in two dimensions it was decided to use as parameters S_1 and S_2, the conductances, i.e. the 'depth-integrated conductivities', of the first and second layers. This provided the two-dimensional S-space of the model.

The only constraints placed on the four variables were that they had to be physically realisable – the layer resistivities had to be in the range $1\ \Omega\text{m} < \rho < 10^4\ \Omega\text{m}$ and the layer thicknesses had to be positive. An 85 per cent acceptance level ($0.85 \times 18 = 15.3$, i.e. the theoretical response had to lie within 16 confidence intervals) was chosen and 10 000 models were randomly selected and their theoretical responses were compared with the observed response. The resulting S-space is shown in Fig. 3(a) and the resistivity–depth ($\rho - d$) profiles of the accepted models in Fig. 3(b). The S-parameters of the models accepted are shown in Fig. 3(a) by black dots. It is apparent from Fig. 3(b) that, for a model to be within 85 per cent of the confidence intervals of the BOR data, the top layer resistivity has to be in the range 250–3000 Ωm and the thickness between 16 and 50 km. The resistivity and thickness of the middle layer have to be in the ranges 10–125 Ωm and 4–70 km respectively. Fig. 3(a) shows that the conductances of the layers of the models have to be $10\ \text{S} < S_1 < 250\ \text{S}$ and $650\ \text{S} < S_2 < 400\ \text{S}$.

It is apparent from Fig. 3(b) that a conducting zone ($\rho < 125\ \Omega\text{m}$) at depth satisfies the

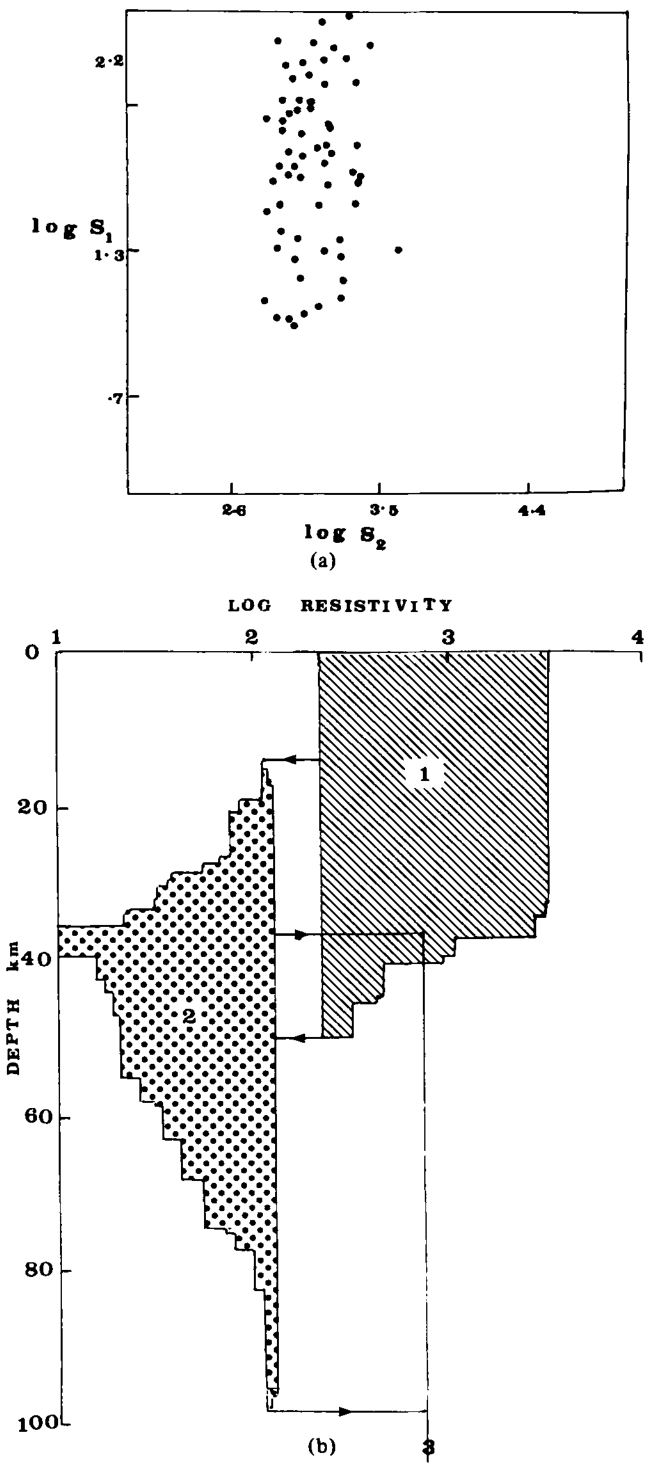
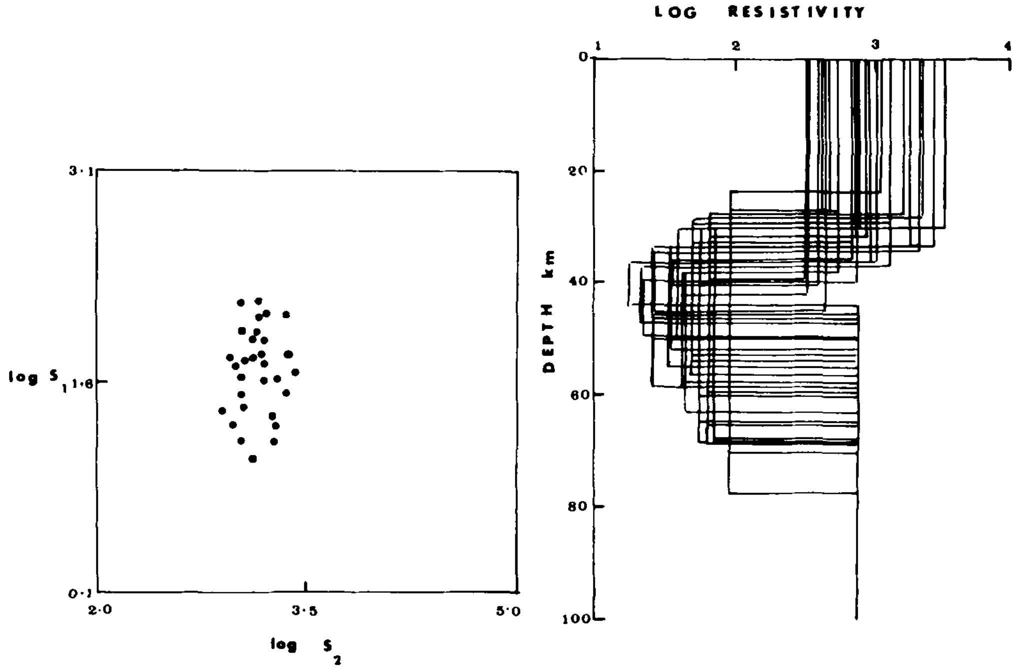


Figure 3. (a) The *S*-space for the Monte-Carlo inversion of data from BOR at the 85 per cent acceptance level. (b) The resistivity–depth profiles of models acceptable to the BOR responses at the 85 per cent acceptance level. For clarity in this diagram, the outermost bounds only for the three layers have been indicated.



(a) (b)

Figure 4. (a) The S-space for the Monte-Carlo inversion of data from BOR at the 94 per cent acceptance level. (b) The resistivity–depth profiles of models acceptable to the BOR responses at the 94 per cent acceptance level.

observed response at station BOR. However, the depth to the top of the zone can take any value in the range 16–50 km. In order to investigate, as before, the effects of a higher acceptance level than 85 per cent, the parameters of those models whose theoretical responses lay within 17 of the 18 confidence intervals (i.e. a 94 per cent acceptance level) were plotted. The S-space of these models is illustrated in Fig. 4(a) and their ρ–d profiles in Fig. 4(b). By comparing Fig. 3(b) and 4(b), it is apparent that use of the higher acceptance level considerably reduces the permitted ranges of the four variables. The effect of the percentage acceptance level on the permitted ranges of the variables is summarized in two tables.

It was possible to attempt to reduce the permitted ranges further by demanding that the theoretical response of an acceptable model passed within all the apparent resistivity

Table 2. Permitted parameter range for BOR.

Percentage acceptance level		85	94	> 94
Resistivity of top layer	ρ_1	250–3000 Ωm	300–3000 Ωm	300–3000 Ωm
Thickness of top layer	h_1	16–50 km	24–45 km	24–42 km
Resistivity of second layer	ρ_2	10–125 Ωm	17–90 Ωm	20–90 Ωm
Thickness of second layer	h_2	4–70 km	8–54 Ωm	10–54 Ωm
Depth of base of second layer	d_2	15–> 100 km	44–77 km	46–77 km
Conductance of top layer	S_1	10–250 S	10–160 S	10–160 S
Conductance of second layer	S_2	650–4000 S	800–2500 S	800–2500 S

confidence intervals and all but one of the phase confidence intervals (column 3 of Table 2), but the improvement was negligible.

This application of the Monte-Carlo inversion procedure appears to have provided some indication of the range of acceptable electrical conductivity models at BOR. The validity of the two basic assumptions which have been made in this illustration – (1) the distribution can be approximated by three layers, and (2) the base layer resistivity is 750 Ωm – has been considered by Jones (1977) and is discussed later. While it may be necessary to modify these assumptions when well-estimated responses become available over a larger period range, it is considered that this application of the Monte-Carlo inversion procedure has provided a useful indication of the range of acceptable conductivity models at BOR.

3 Monte-Carlo inversion of the MT results from southern Scotland

In this section the inversion of the MT results by the above Monte-Carlo method is discussed. Each of the three regions – the Midland Valley, the Southern Uplands and northern England – is considered in turn, in view of the similarity of the response functions of stations within the same region.

3.1 MIDLAND VALLEY RESULTS – FTH AND SAL

The FTH and SAL resistivity and phase data which were inverted in the manner discussed in Section 2, are given in part I fig. 7(a) and (c). For station FTH, with 'well-estimated' data at five periods between 50 and 800 s, there were 10 confidence intervals, five for apparent resistivity and five for phase. At 50 s, the interval for the phase ranges from 0 to 90°, thus satisfying any conductivity model. Hence there were only nine intervals which could be used to determine the acceptability of a model. The Monte-Carlo inversion procedure was initially applied to the data with an acceptance level of 80 per cent, with a three-layer conductivity profile, whose response was similar to the observed response, being used as the starting model and the base layer resistivity being held constant at 750 Ωm. From a total of 20 000 models randomly selected from the four-dimensional parameter space, those models which are accepted at an 80 per cent level were re-examined to determine those permitted also at a 90 per cent level. The $\rho-d$ profiles of those models accepted at the 80 per cent level are shown schematically in Fig. 5(a) and the parameter ranges for the two levels of acceptance in Table 3.

There were 11 impedances considered well-estimated for station SAL and the period range was from 28.5 to 600 s, but for reasons discussed by Jones (1977) it was considered that only 19 of the 22 confidence intervals could be used to determine acceptable models.

The SAL data were inverted, in exactly the same manner as the FTH data, and the $\rho-d$ profiles of the models accepted at the 75 per cent level (i.e. a minimum of 16 of the 19 intervals) are illustrated schematically in Fig. 5(b). The resemblance between Fig. 5(a) for FTH and 5(b) for SAL is striking, but perhaps not surprising considering that the two data sets are similar. The maximum depth to the conducting zone is 13 km and the conducting zone resistivity is in the range 35 Ωm $< \rho_2 <$ 80 Ωm. The base of the zone is at a depth of 33–80 km.

When the acceptance level is increased to 77 per cent, the permitted models have a depth to the second interface, i.e. the base of the conducting zone, of $44 < d_2 < 67$ km. The resistivity of the conducting zone is in the range 35 Ωm $< \rho_2 <$ 70 Ωm and its top interface is at a depth less than 12 km.

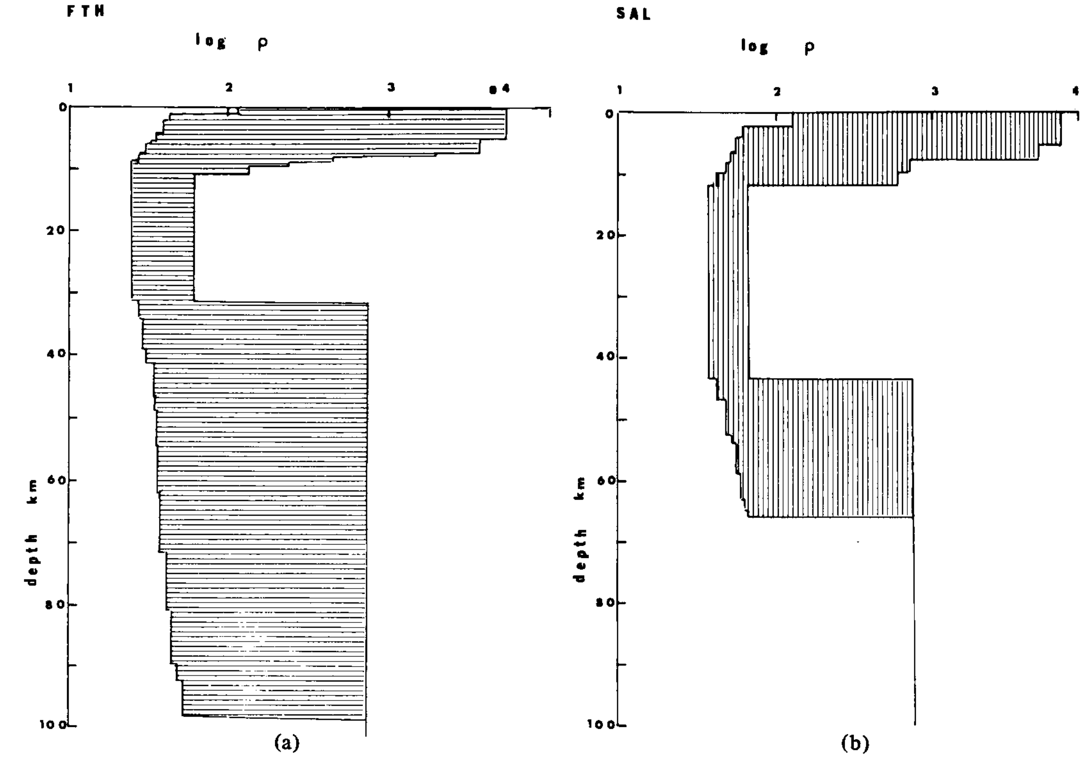


Figure 5. (a) Outline of resistivity–depth profiles for FTH rotated major data at the 80 per cent acceptance level. (b) Outline of resistivity–depth profiles for SAL rotated major data at the 75 per cent acceptance level.

Table 3. Permitted parameter ranges for FTH.

Percentage acceptance levels		80	90
Resistivity of top layer	ρ_1	100–5500 Ωm	100–5500 Ωm
Thickness of top layer	h_1	< 11 km	< 11 km
Resistivity of second layer	ρ_2	20–70 Ωm	25–60 Ωm
Thickness of second layer	h_2	> 20 km	> 25 km
Depth of base of second layer	d_2	> 27.5 km	> 32 km
Conductance of top layer	S_1	0.3–110 S	0.5–110 S
Conductance of second layer	S_2	650–2000 S	800–2000 S

3.2 SOUTHERN UPLANDS RESULTS – BOR, NEW AND PRE

In part I, Jones & Hutton justified a one-dimensional treatment of the rotated major data for BOR, NEW and PRE in the Southern Uplands.

In exactly the same manner as described in Section 2 for BOR, the NEW and PRE results were inverted by the Monte-Carlo procedure assuming in each case a 3-layer model with a base layer of resistivity 750 Ωm. Their MT responses are given in fig. 9 (part I) and their ρ–d profiles are illustrated in Figs 6(a) and 7, the acceptance levels being 90 per cent for NEW and 94 per cent for PRE. The data from station NEW were also inverted assuming an 8-layer model with seven layers of fixed thickness 10 km. The 90 per cent acceptance

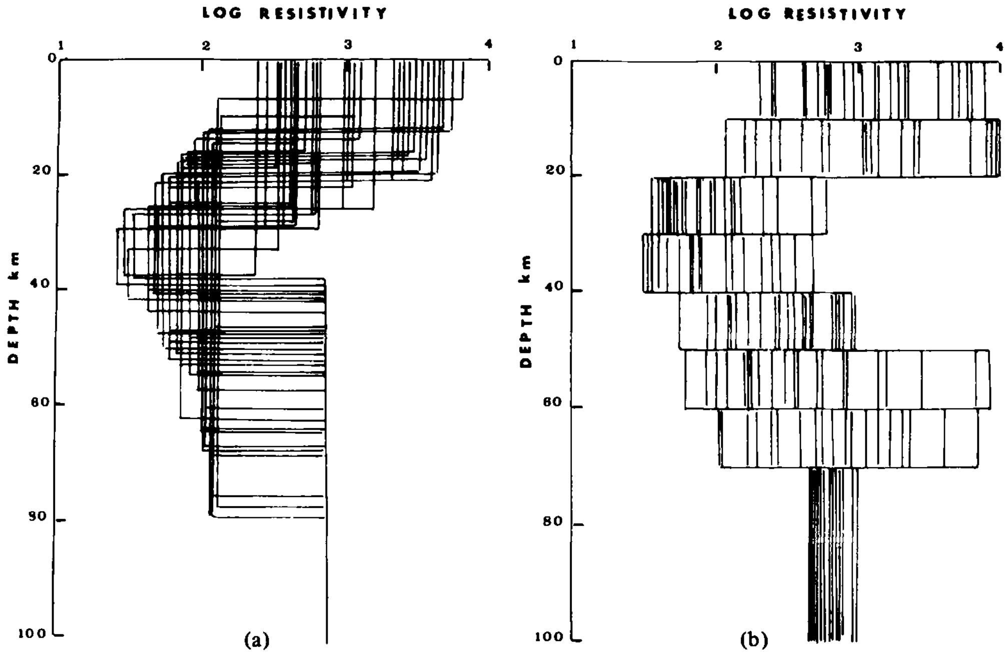


Figure 6. The resistivity–depth profiles for NEW rotated major data at the 90 per cent acceptance level. (a) 3-layer models. (b) 8-layer models.

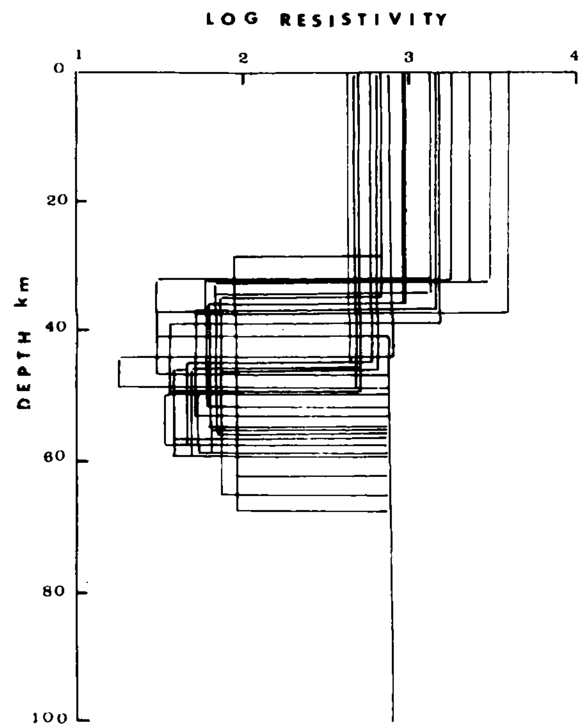


Figure 7. The resistivity–depth profiles for PRE rotated major data at the 94 per cent acceptance level.

models for this procedure are displayed in Fig. 6(b). This inversion indicates (1) that a 3-layer model of the form resistive layer/conductive layer/resistive layer adequately satisfies the data, and (2) that the base layer resistivity is confined to the bounds 500 Ωm < ρ_{base} < 1000 Ωm. Hence, the two basic assumptions of a 3-layer model with a base layer of 750 Ωm appear justifiable.

The maximum resistivity of the conducting zone under PRE was found to be 90 Ωm (at the 94 per cent level) which is the same as for BOR. Hence, there is a strong case for assuming that 90 Ωm will be the maximum resistivity of the zone beneath NEW as well.

3.3 NORTHERN ENGLAND RESULTS – TOW

For the MT data from TOW – fig. 11 (part I) – the only location in northern England at which MT variations were recorded, there are nine impedance estimates giving 15 model acceptability constraints (three of the phase intervals span the range 0–90°). The ρ–d profiles of three layer models, with base layer resistivity of 750 Ωm and an acceptable response at the 89 per cent level (within 11 of the 15 intervals) are illustrated in Fig. 8. It is apparent from the inversion that (a) the resistivity of the top layer has negligible effect, (b) there is a highly conducting layer, of resistivity 3 Ωm < ρ < 11 Ωm, in the upper crust under TOW, and (c) the conductivity variations at the base of the crust and in the upper mantle are not resolved by the data.

As it seemed possible that the highly conducting (10 Ωm) upper layer at station TOW may have 'screened' the effect of an underlying conductive layer, further inversion was undertaken to determine if the conductive zone beneath the Midland Valley and the

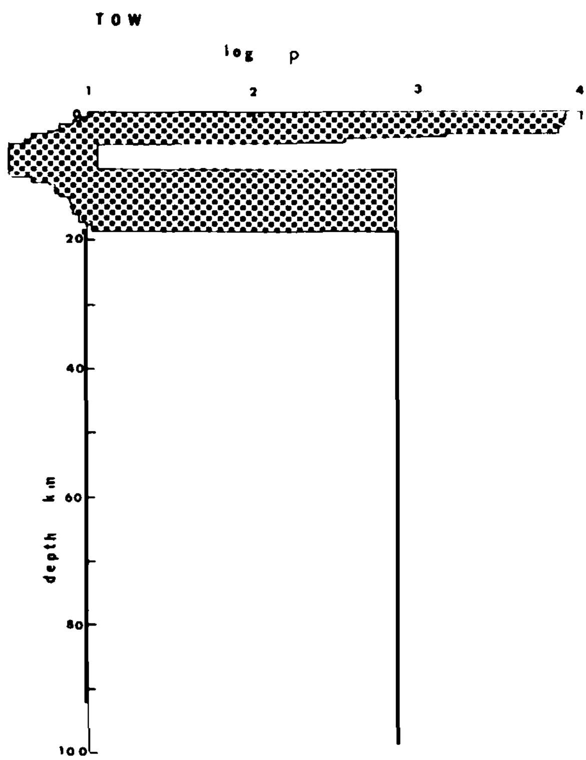


Figure 8. The outline of resistivity – depth profiles for TOW in northern England.

Table 4. Permitted layer resistivities and thicknesses for a 3-layer model with a base layer of 750 Ωm.

	First layer		Second layer		Depth to basement
	ρ_1	h_1	ρ_2	h_2	d_2
Midland Valley sites					
FTH	100–5500	< 11	25–60	> 21	> 32
SAL	125–7500	< 13	35–70	32–63	44–67
Southern Uplands sites					
BOR – maj	300–3000	24–42	20–90	10–54	46–77
min	250–3250	18–50	20–90	11–58	42–76
NEW*	250–6300	16–37	25–90	9–46	37–62
PRE	400–4000	28–50	17–90	4–40	40–68
Northern England site					
TOW	120–8000	0.5–5	3–11	9–19	5–19

Southern Uplands could also be present under northern England. For the inversion procedure, the resistivities of the 3-layer model were kept constant, at $\rho_1 = 10\ \Omega$m, $\rho_2 = 50\ \Omega$m and $\rho_3 = 750\ \Omega$m, and the two-layer thicknesses, h_1 and h_2, were permitted to vary randomly. This study indicated that if a 50 Ωm layer does underlie the 10 Ωm zone, then the model parameters are constrained by: $\rho_1 = 10\ \Omega$m, 8 km < d_1 < 13 km, $\rho_2 = 50\ \Omega$m, 28 km < h_2 < 40 km and 28 km < d_2 < 40 km.

4 Geophysical interpretation

This quantitative interpretation is restricted to the results of the one-dimensional inversion of the MT data, discussed in Sections 2 and 3. The permitted ranges of the parameters of the models are summarized in Table 4 and Fig. 9.

It is apparent from Table 4 and the geoelectric section that there must be a conducting zone, of resistivity 35–60 Ωm, underlying the Midland Valley at a depth no greater than about 12 km. The zone must be at least 32 km thick and it must extend to a depth greater than 44 km. Hence, this zone occupies all the 'lower crust' and the top part of the 'upper mantle'. The terms 'crust' and 'mantle' are used here to aid description of features above and below the arbitrary depth of 30 km. The base of the conducting zone is not well resolved by the data. There may also be an additional surface conductor in this region due to the Carboniferous sediments but as shown in part I (fig. 12), the data are unable to resolve the shallow structure.

There is also a conducting zone underlying the Southern Uplands, with a resistivity of 25 Ωm < ρ < 90 Ωm. Under BOR and PRE, which lie on a line of geological strike, this zone is at a depth greater than 28 km. Therefore, the crust under these sites is resistive and the upper mantle conductive. There is some evidence that the conducting zone may become shallower to the south-east because its top interface is permitted to be as close at 16 km to the surface under NEW. The maximum permitted depth of the base of the zone under all three of these stations is of the order of 65–75 km.

Whether the conducting zone under the Midland Valley is due to the same conditions that cause the low resistivities under the Southern Uplands cannot be confirmed, or otherwise, from this analysis. However, there is some probability that the two are related because of their similar resistivities.

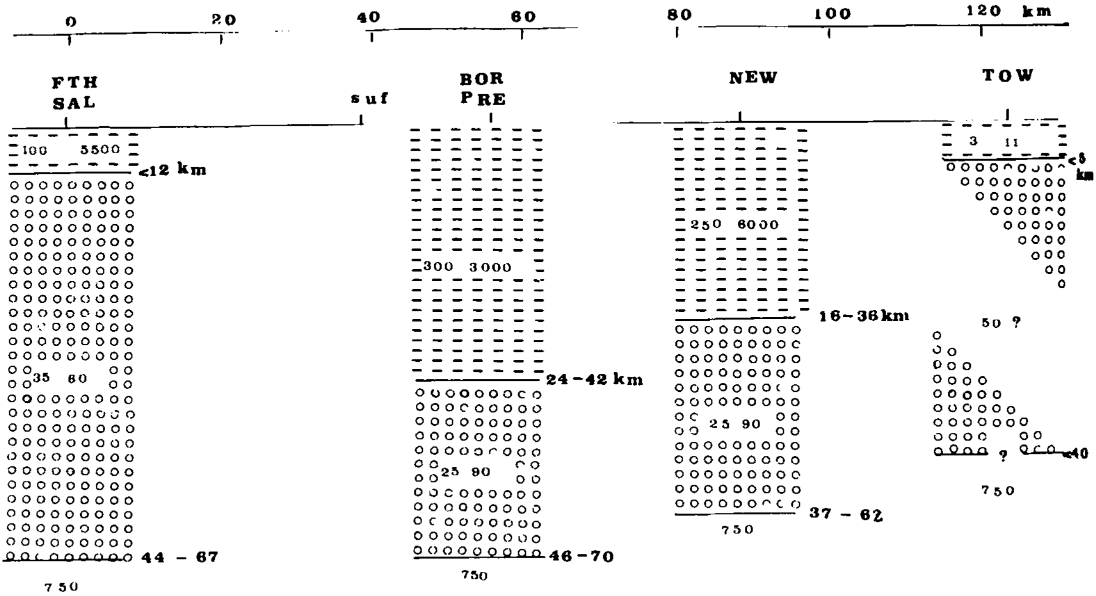


Figure 9. A schematic geoelectric section of the Southern Uplands. The distances along the traverse and the depths of the layer boundaries are in kilometres and resistivities in ohm metres. SUF is the Southern Uplands Fault.

There are many possible causes for low resistivity zones at lower crustal/upper mantle depths. The explanation normally given for a conducting zone in the crust is based on hydration processes, with or without associated partial melting. Since a temperature of more than 950 K is required to partially melt rocks under water-saturated conditions in the crust, partial melting at the crust–mantle interface generates a geothermal gradient in the crust in excess of 20 K/km. This would normally result in an above average heat flow. Unfortunately no reliable heat flow measurements are yet available for this region – those, which have been made, may not be reliable as they have been made in coal mines. They indicate only modest heat flow values of 1.3–1.5 hfu (Garnish 1976). However, since a conducting zone exists up to 12 km from the surface, a geothermal gradient of greater than 60 K/km would normally result if partial melting were responsible for the low resistivities at these shallow depths. Such a high gradient would be expected to generate hot springs, as in Iceland, but these are not observed. It thus seems likely that hydration processes are largely responsible for the conducting zone at shallow depths under the Midland Valley.

It is possible, on the other hand, that partial melting of the hydrated rocks could be responsible for the observed conductors at upper mantle depths under both the Midland Valley and the Southern Uplands. A partial melt at this depth might also explain why Bamford & Prodehl (1977) did not observe clear arrivals from the Moho when they analysed the LISPB seismic data for the region (Bamford *et al.* 1976).

The conducting zone under northern England, as detected by the data from TOW, is very different in character from that under the other stations. It is apparently in the upper crust and has a conductivity which is an order of magnitude greater than that of the other conducting zone(s). As discussed in Section 3.3, it is possible that this zone could 'screen' an underlying layer of similar resistivity to that in the upper mantle under the Midland Valley and the Southern Uplands, i.e. 20–90 Ωm. The most likely explanation for the highly conducting zone ($\sigma \sim 0.1$ S/m) in the upper crust is electrolytic conduction in the water-saturated sediments which infill the Northumberland Basin, which is known to be deeper than 3 km.

Since well-estimated resistivities and phases for periods up to 2000 s were obtained from the NEW records, they were re-examined to discover if the data resolved any of the olivine phase transformations suggested by the laboratory work of Yagi & Akimoto (1974). A three-layer model, with a theoretical response close to the observed response, is illustrated, together with its response and the data from NEW, in Fig. 10 (model 1). The responses of three further models, with their upper layers as for model 1 but with a base layer of 1 Ωm at 400, 600 and 800 km, were then calculated and compared with the observational data. It is apparent from the computed responses – Fig. 10, curves 1, 2 and 3 – that *the well-estimated data* (those plotted with asterisks) cannot resolve the proposed deep structure. If, however, *all* the estimates are considered, it appears that the transition to a high conducting zone must be at some depth greater than 800 km. A conducting zone at such depths satisfies both the amplitude and phase long-period responses. A gradational transition over 200 km, suggested as the maximum transition width by Banks (1969) would also be acceptable to the observed response.

5 Tectonic implications

In recent years, there has been much discussion by geologists (e.g. Phillips, Stillman & Murphy 1976) concerning the association between the tectonic history of southern Scotland and that of eastern Canada. It is thus interesting to compare the results of this study with those of a magnetotelluric study in eastern Canada (Kurtz & Garland 1976). A major result

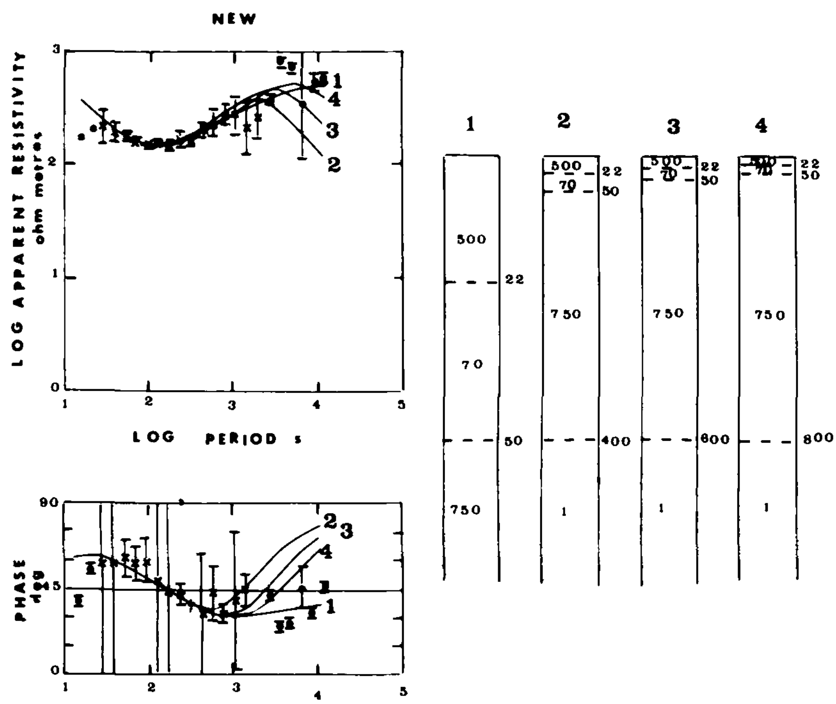


Figure 10. A comparison of 3 and 4 layer theoretical models for NEW. The apparent resistivity and phase curves for each of the models (b) is superimposed on the apparent resistivity and phase values plotted in (a). The model depths are in kilometres and resistivities in ohm metres.

of the Canadian study was the contrast in conductivity structure between the Shield and Appalachian regions. In the former, there is a good conductor in the crust while the mantle is resistive and in the latter, the crust is resistive and there is a conductor in the uppermost mantle (Kurtz & Garland 1976, fig. 22). In particular, the interpretation of the responses from their station 10 in the Shield region is directly comparable to the accepted models for the structure between stations FTH and SAL of this study, and those of their station 14 in the Appalachian region to those of the structure beneath the Southern Uplands, i.e. at stations BOR, NEW and PRE.

Hence, the conductivity variations beneath eastern Canada can be strongly correlated with those beneath the Midland Valley and the Southern Uplands of Scotland. Since the Canadian 'station 14' lies directly on the extension of the Reach Bay fault line – recently suggested by McKerrow & Cocks (1977) as the Iapetus suture zone in Newfoundland – there is good evidence from the comparable electrical conductivity data for proposing that the Iapetus suture zone in Britain is now represented by the Southern Uplands.

6 Conclusions

This magnetotelluric study has shown that there is a significant variation in electrical conductivity structure in the region between the Midland Valley of Scotland and northern England. In this respect, it confirms the interpretation of geomagnetic deep sounding data from this region. Presentation of these data using induction arrows and the hypothetical event technique (Hutton & Jones 1978) located two narrow belts across which there must be marked lateral changes in electrical conductivity structure. One of these was parallel to the Southern Uplands Fault but slightly to the south of it and the other was near the Scottish–English border and had an east–west strike. Application of the MT method has now given a quantitative geo-electric section of the three regions separated by these two belts. It thus complements the GDS techniques which are particularly useful in delineating the lateral variations in structure, by providing resistivity versus depth profiles for the three regions as follows:

(1) Under the Midland Valley, there is a conducting zone at a depth of *no greater than* 12 km. The whole of the lower crust, and possibly part of the upper mantle, under stations FTH and SAL appears to be of low resistivity ($35\ \Omega\text{m} < \rho < 60\ \Omega\text{m}$).

(2) Contrary to the conclusions of Jain & Wilson (1967), there is a strong suggestion that the conducting zone underlying the Southern Uplands is at a depth *greater than* 24 km. There is some evidence that the top interface of the zone might become closer to the surface to the south-east, i.e. under NEW. The permitted resistivity of the zone is in the range 29–90 Ωm, which is comparable to the permitted resistivity of the zone under FTH and SAL. Whether the two are due to the same dominant conducting mechanism cannot be evaluated from this study, but the similarity in their resistivities gives some weight to such a supposition.

The most likely explanation for the conductors is hydration processes in the crust and upper mantle. Associated partial melting is probable at upper mantle depths. These effects should not be assumed exclusively however, as many other conducting phases could give low resistivity in the upper mantle (Duba 1976) and metamorphic rocks, such as graphitic schists, could give low resistivity zones in the crust.

(3) Under northern England, there is indication of a very highly conducting zone ($0.1 < \sigma < 0.35$ S/m) very close to the surface. This zone is probably due to the conducting sediments which fill the Northumberland Basin. The conducting processes would thus be dominated by electrolytic conduction in the pore fluid, as given by Archie's Law (Archie

1942). For rocks of conductivity 0.1 S/m, and probable pore water conductivity of 1 S/m (Keller 1966), the required porosity is from 0.1, for $m = 1$, to 0.3, for $m = 2$. This porosity range is acceptable for most limestone and sandstone formations (Keller 1966), such as are found with great thicknesses in the Northumberland Basin (Eastwood 1953). This conducting zone may 'screen' an underlying layer which is the southward continuation of the conducting zone beneath the Midland Valley and the Southern Uplands.

A general conclusion, to which attention is drawn, is that modelling studies show clearly the importance of estimating the phase as well as the amplitude response. For example, it was possible to satisfy the amplitude data alone from PRE without a conducting layer but the phase response for such a model was totally unacceptable. Also the phase response gives better resolution of the surface structure, as shown by Summers (1976).

Further study of this region is now in progress.

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